Exercise 11

Prove the identity.

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$$

Solution

Use the definitions listed on page 259.

$$\begin{aligned}
\sinh x \cosh y + \cosh x \sinh y &= \left(\frac{e^x - e^{-x}}{2}\right)\left(\frac{e^y + e^{-y}}{2}\right) + \left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^y - e^{-y}}{2}\right) \\
&= \frac{(e^x - e^{-x})(e^y + e^{-y})}{4} + \frac{(e^x + e^{-x})(e^y - e^{-y})}{4} \\
&= \frac{(e^x - e^{-x})(e^y + e^{-y}) + (e^x + e^{-x})(e^y - e^{-y})}{4} \\
&= \frac{(e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y}) + (e^{x+y} - e^{x-y} + e^{-x+y} - e^{-x-y})}{4} \\
&= \frac{2e^{x+y} - 2e^{-x-y}}{4} \\
&= \frac{e^{(x+y)} - e^{-(x+y)}}{2} \\
&= \sinh(x+y)
\end{aligned}$$